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Self-organisation and fracture connectivity in rapidly heated continental crust

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Abstract—Volume expansion (~1–5% volume strain with $\Delta V_{\text{melting}}$ positive) and fluid-absent partial melting, in which $\Delta V_{\text{melting}}$ is positive, of continental crust by intruding basaltic magma is a strongly irreversible process involving the dissipation of both thermal energy and matter (partial melt). Using a simple random graph model we show by analogy how isolated fractures that form during rapid thermal perturbation in the source region can combine to form a single, interconnected structure with high permeability. Once connected, the fracture network may be thought of as a single structure or pattern that will remain stable so long as a strong temperature gradient is maintained in the source region. Estimates of fracture permeability that take into account changes in connectivity and fracture spacing range from approximately 10^{-10} to 10^{-5} m^2, many orders of magnitude greater than values considered typical during large-scale crustal deformation and prograde regional metamorphism. The ability of the isotropic fracture network to develop a top–bottom directionality is crucial for buoyancy-driven melt transport. A physical model based on non-linear evolution rules during thermal expansion is given that predicts the emergence of directionality (vertical fracture alignment) on a time scale of the order of 10^5 y. The necessary ingredients are a deviatoric strain path, a heterogeneous medium and a stiffness that evolves as a function of the local strain. © 1998 Elsevier Science Ltd. All rights reserved

INTRODUCTION

Ductile rocks exceed their elastic limit if changes in volume take place faster than deformational strain can be accommodated by creep (e.g. Shaw, 1980). Tensile failure resulting initially from sudden volume changes and/or melt-enhanced embrittlement (Rutter and Neumann, 1995) will be especially effective where partial melting is fast, as is generally the case during intrusion of mafic magmas into easily fusible crustal rocks (Huppert and Sparks, 1988). Rapid thermal perturbation and partial melting of the continental crust by intruding mantle-derived magmas is a non-reversible process, characterised by large thermal and rheological gradients over small length scales (Bergantz, 1989; Petford, 1995). Volume changes in the crustal source region during underplating/intraplating of basaltic magma occur as a result of both thermal expansion and partial melting. It has long been known that the presence of an interstitial fluid phase in crustal rocks can lead to a dramatic reduction in rock strength (e.g. Hubbert and Rubey, 1959), and that for fluid pressures approaching the total confining (lithostatic) pressure, failure will take place in a brittle manner. Where partial melting occurs in the absence of a free fluid-phase (fluid-absent melting), positive volume changes of ca. 2–20% may also lead to enhanced local fracturing of the protolith (e.g. Clemens and Mawer, 1992; Petford, 1995; Connolly *et al*., 1997).

These processes have important bearing on the long-standing problem of how granitic partial melt segregates from its matrix. Whilst the majority of field evidence suggests that this process occurs primarily by fracturing of the protolith (Brown, 1994), opinion is divided as to whether the fractures form predominantly during regional deformation contemporaneous with partial melting (e.g. Brown *et al*., 1995; Collins and Sawyer, 1996), or locally in direct response to the partial melting process (Petford, 1995).

In this paper we argue that during the initial stages of heating and partial melting, the crustal protolith is driven far from thermal and mechanical equilibrium with respect to its immediate surroundings—to the extent that structures emergent within it (e.g. fractures) will self-organise. The fracture network forms because of volume changes as the protolith overlying the intrusion is heated up and expands under high effective stress. As the protolith is driven further from thermal equilibrium, fractures grow and eventually link up as the protolith attempts to lose (dissipate) thermal energy. At this point, the fracture network can be considered a single, interconnected, more or less coherent structure or pattern. Using a simple random graph model to simulate fracture formation in homogeneous, partially molten crust, we show by analogy how the architecture (connectivity and permeability) of the system can evolve into a discrete, structured conformation with a high degree of fracture connectivity—a crucial step in allowing melt to segregate effectively from its matrix. A general fracture permeability model that incorporates connectivity is then used to derive general estimates of local permeability in the region of partial melting.

The difficulty with self-organisation theory is that the evolution rules of the system have to be supplied phenomenologically and are not part of the conceptual framework of the theory itself. The underlying physical mechanisms are thus not addressed, only the outcome of assumed process parameters can be calculated. This is addressed by putting forward a physical model (Koenders, 1997) in which the rock is regarded as a heterogeneous medium where a fluctuational stiffness field is introduced to mimic the fractured protolith. The protolith then undergoes an evolution based on the isotropic strain increment, with the condition that as the rock expands in a certain location, the stiffness will be reduced and if the rock becomes more dense the Young's modulus will increase. A calculus based on the mechanical equilibrium equations relevant to quasi-static mechanics can describe the distribution of strain throughout the protolith during heating. The physical model predicts the development of coherent directional structures (fractures) analogous to the self-organisation in the connectivity model. Thus a link is made between the discrete approach, which cannot make use of mechanical laws (but which has the virtue of simple visualisation) and a consistent physical mechanism for the evolution of fracture connectivity based on continuum mechanics.

SELF ORGANISATION

Self-organisation appears to be a fundamental aspect of all natural systems driven far from equilibrium by a constant energy flux. Geometrically ordered structures such as layering and pattern formation arise spontaneously (e.g. Nicolis, 1977; Coveney and Highfield, 1995; Bak, 1997) and are governed by apparently simple, low-level rules (e.g. Kauffman, 1995). The driving force for self-organisation in natural systems is the dissipation of energy and matter (Prigogine, 1980).

Given that the majority of physical (and biological) processes take place out of equilibrium, self-organisation seems likely in all non-linear dynamic systems at some stage during their life-time (Xi *et al.*, 1993). A familiar geophysical example is the hexagonal arrangement of convection cells (Rayleigh–Benard convection) that develop in thermally perturbed fluids (Bodenschatz *et al.*, 1991). Other examples of self-organisation in the solid earth, which, together with self-similarity and $1/f$ noise are referred to generically as self-organised criticality (Bak *et al.*, 1988), include fault and fracture populations (Turcotte, 1992), earthquakes (Ito and Matsuzaki, 1990), sand piles and avalanches (Freete *et al.*, 1996), ice sheets (Payne and Dongelmans, 1997), turbidite deposits (Rothman *et al.*, 1994), river drainage (Rinaldo *et al.*, 1993) and evolutionary fluctuations in the fossil record (Sole *et al.*, 1997).

An important non-linear process, which has direct relevance to this study, is percolation. The theory of percolation defines certain critical thresholds or points that mark sudden phase transitions (e.g. Stauffer, 1985). A good example occurs in cement slurries, where at some critical concentration, isolated clusters of suspended particles join up to form an interconnected solid that spans the entire system. In the following section we expand on this theme and show by analogy how a highly connected, self ordered (fracture) network might develop gradually from an initially random set of nodes and lines in accord with simple rules common to many non-linear systems.

CONNECTIVITY MODEL

Figure 1 is a two dimensional graphical simulation showing how a simple fracture network might, in principle, evolve in partially melted crust. It is based on a random graph model (Kauffman, 1995), and consists of a set of 100 random dots (nodes) connected by lines. Consider that any two nodes in Fig. 1(a) mark a fracture tip, with a single fracture represented by a line in between. By simply joining up the dots at random, it is possible to 'evolve' the simulated fracture system through increasing levels of connectivity. The changing connectivity (f) is defined by the ratio of connected to unconnected nodes (c/n) over the interval $0 \leq f \leq 1$.

Coupled to increasing connectivity is the number of largest interlinked clusters of lines (N). At low connectivities N is small, but becomes large as connectivity increases. The transition between unconnected (isolated) and highly connected nodes occurs over a relatively small range in connectivity, often close to a c/n ratio of 0.5, and is a fundamental property of all random graphs (Kauffman, 1995). Taking Fig. 1 as an example, at 25% (c/n = 0.25) connectivity, the largest cluster (N) is two. However, at 50% connectivity (c/n = 0.50), the largest simulated cluster of fractures has jumped to 12 (Fig. 1c). At 75% connectivity however, the largest cluster has grown by only two to 14 (Fig. 1d).

The change in cluster size (N) with increasing connectivity (c/n) is shown in Fig. 2. As expected, the curve is sigmoidal, showing low cluster growth at low values of connectivity ($0 \leq f \leq 0.2$), increasing rapidly through a transition region (f = 0.3–0.6) and stabilising at $f \geq 0.6$. At connectivities in excess of 60%, the majority of nodes and lines are in communication with one another. The network is in effect a single structure, or 'supercluster' (Kauffman, 1995). Applying this model to the crust, we can predict that connectivity, and hence the efficiency of melt segregation, will increase dramatically across the transition region to a maximum when the fracture system forms a supercluster. The transition region, defined as the steepest part of the curve, is characteristic of non-linear systems that display avalanche-like behaviour.

In the following section we show how the degree of (imaginary) self-organised connectivity can be related directly to the evolution of fracture permeability in real world systems.

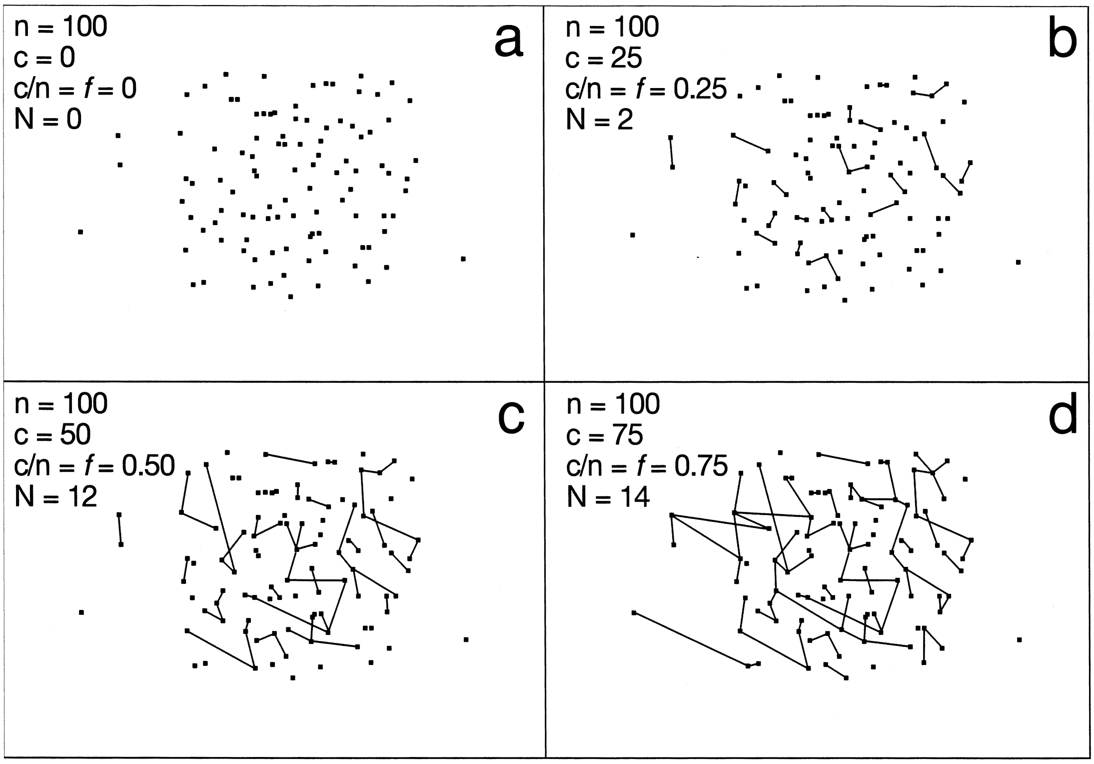

Fig. 1. Random graph model showing the evolution of an initially random configuration of 100 nodes for an imaginary fracture network that forms during progressive crustal melting. (a) Unconnected. (b) Twenty-five nodes connected, connectivity ratio (c/n) = f = 0.25. Largest connected cluster (N) = 2. (c) Connectivity ratio = 0.5, largest connected cluster (N) = 12. (d). Connectivity ratio 0.75, largest cluster (N) = 14.

CONNECTIVITY–PERMEABILITY MODEL

The development of fracture permeability is a crucial process that controls both the degree of melt segregation and subsequent large-scale transport of granitic melt out of the source region. Accurate modelling of this process requires an understanding of both the macroscopic properties of the protolith matrix (e.g. grain size, porosity, fracture aspect ratio and fracture spacing) and the amount and physical properties

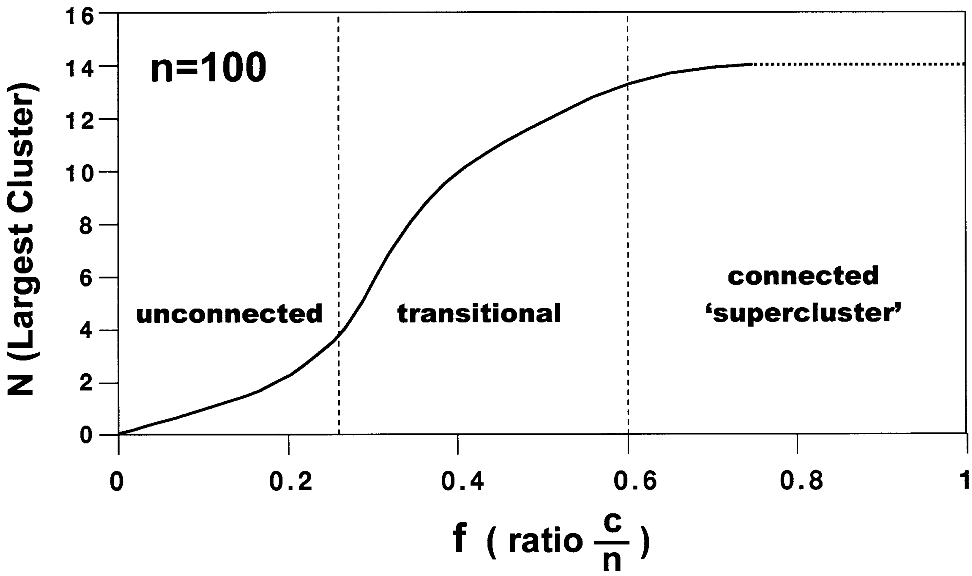

Fig. 2. Plot showing the degree of connectivity, expressed as the largest individual cluster (N), as a function of connectivity ratio (f) for the random graphs shown in Fig. 1. The S-shaped curve is typical of avalanche dynamics in non-equilibrium systems. The gradient of the curve in the transition region depends upon the number of nodes, and will steepen as this number increases. N increases rapidly at 30% connectivity, stabilising at around f = 0.6. At $f \leq 0.3$, most nodes remain unconnected. At $f \geq 0.6$, the majority of nodes are linked together into a coherent structure.

(viscosity and density) of the partial granitic melt (see Brown *et al.*, 1995).

Melt viscosity and matrix permeability limit the flux of melt out of the source region. Given that rates of fluid flow through a continuous medium of fixed permeability vary with the inverse of fluid viscosity (e.g. Dullien, 1979), a high degree of permeability is advantageous during segregation of relatively viscous (10^3–10^6 Pa s) granitic melts (Petford, 1995). The physical evolution of partial melt with regards to its solid matrix may usefully be thought of as a process of ripening. A melt will be 'ripe' (e.g. lowest possible viscosity and maximum gravitational instability), when its viscosity and density are such that its ability to escape its surroundings is maximised. However, no matter how ripe the melt phase, extraction will not occur unless the matrix is sufficiently permeable. It is stressed that it is fracture *connectivity*, not simply the existence of fractures that largely controls the fate of crustal melts through increased permeability in the source region. Using percolation theory, Stauffer (1985) and Dienes (1982) have shown that connectivity (defined statistically as the probability of fracture intersection) exerts a fundamental control on permeability. These (and many other) percolation models share a common feature referred to as the percolation threshold, characterised by a sudden increase in permeability over a narrow range in connectivity. Similar behaviour is seen in the random graph model above, where a narrow transition region separates isolated and highly connected fractures.

A useful way to assess the relationship between increasing connectivity (f) and permeability (k) is given by Gueguen and Dienes (1989) as:

$$k = \frac{4\pi}{15} A^3 n_0 c^5 f \quad (1)$$

where A is the fracture aspect ratio, n_0 the fracture number density $1/l^3$ (where l is the fracture spacing), and c is the fracture length. By taking f (number of connected fractures) in equation (1) as equivalent to the ratio c/n in Fig. 2, we can make some assessment of the sensitivity of f on fracture permeability in a hypothetical source region. By way of example, the change in permeability in a fracture system with an arbitrary mean spacing of 0.1 m and 0.5 m, and fixed aspect ratio of 0.1 is shown in Fig. 3. As a general point, it is worth noting that permeabilities of from ca. 10^{-10} to 10^{-5} m^2 estimated this way are up to 14 orders of magnitude greater than those thought to accompany regional metamorphism (e.g. from 10^{-18} to 10^{-20} m^2; Yardley, 1986).

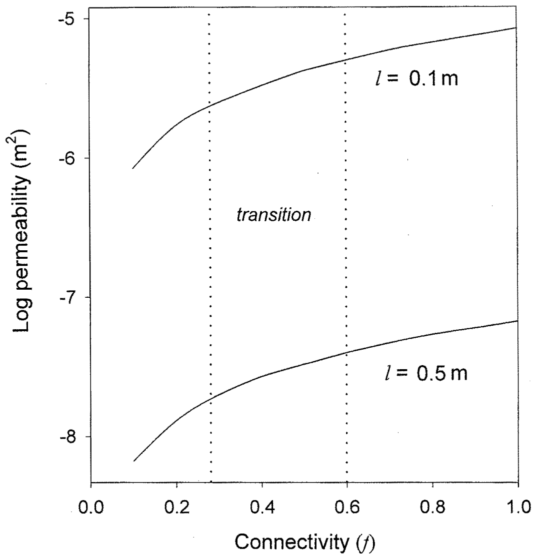

Fig. 3. Plot of log fracture permeability (k) as a function of connectivity (f) for a two-dimensional isotropic fracture network with a mean fracture spacing of 10 and 50 cm and a constant fracture aspect ratio of 0.1. The curve predicts a constant increase in permeability from $\sim 8 \times 10^{-7}$ m^2 at $f = 0.1$ to a maximum of $\sim 8 \times 10^{-6}$ m^2 at $f = 1$ ($l = 0.1$ m) and $\sim 7 \times 10^{-9}$ to 7×10^{-8} m^2 ($l = 0.5$ m). The transition region relates to the sharp increase in fracture connectivity shown in Fig. 2.

PHYSICAL MODEL

The physical model differs from the self-organised connectivity model above in that the protolith is now treated as a *heterogeneous* medium with a fluctuating position dependent stiffness. As the protolith undergoes a strain increment during thermal expansion, the associated deformation will also be position dependent. 'Stiff' bits in the medium will generally deform less than 'soft' bits (a good approximation to the mechanical behaviour of real rocks), with the stiffer elements attracting more stress. If there is a change in the mechanical properties of the protolith associated with the local (thermally induced) strain, then the stiff regions will undergo a smaller change than the softer areas. The result is that the stiffer parts of the material will become relatively more stiff, while the soft parts will become relatively more soft. In addition there are so-called non-local effects so that the region in the vicinity of a soft spot will attract deformation to ameliorate the softness, while the opposite occurs for a stiffer portion.

These effects, first captured in a mode of calculus introduced by Kröner (1967), have been modified by Koenders (1997) to take into account strain increments for simple evolution rules. Evolution—defined here as a change in a system parameter (e.g. stiffness) in response to a change in a state parameter (e.g. volume strain)—results in an effect called 'structures formation' where, statistically speaking, both the stiff and soft areas align themselves as overall deformation goes on, to form coherent spatial structures. These structures are ubiquitous, and arise spontaneously in any system where evolution (i.e. non-linearity, for example plasticity) and heterogeneity occur together, and have particular relevance to granular materials. The structures form gradually, much like a diffusive process, in which the pattern of heterogeneity can be captured in a differential equation and solved (e.g. Koenders, 1997).

The physical model takes a continuum approach, which has the advantage that mechanical equilibrium equations are easily written down. The disadvantage is that, unlike the discrete, self-organised connectivity model discussed earlier, there is no simple, direct way of showing how these continuum structures acquire their directionality. In order to bridge the gap between these two approaches a visualisation of the continuum method is put forward here based on the statistical quantities with which the continuum theory naturally works. These statistical quantities need to be defined rather carefully.

Statistical measures

The distribution of any fluctuating quantity $q(x)$ can be described by its correlation function $\Phi(y)$. The definition of this function involves summing over all products of the value of $q(x)$ taken at two different points and doing this for all points in the interval on which $q(x)$ is defined. Instead of a sum an integral may be taken, thus:

$$\Phi(y) = \frac{1}{x_1 - x_0} \int_{x_0}^{x_1} q(x) q(x+y) \mathrm{d}x \tag{2}$$

Any systematic pattern in the fluctuating quantity is evident in the correlation function, but purely random effects are suppressed. Any periodic pattern becomes visible as a sinusoidal effect with the wavelength of the oscillating pattern corresponding to the statistical periodicity in the noise represented by $q(x)$. When it comes to periodic patterns it is easier to represent the content of $q(x)$ by the Fourier transform of $\Phi(y)$ in a quantity known as the harmonic intensity $S_q(k)$, where k is the wave number variable. A peak in $S_q(k)$ at, say, k_0 corresponds to a periodic pattern with wavelength $2\pi/k_0$.

Evolution

The theory that describes the formation of fracture alignment in the protolith works with a position dependent stiffness, represented by its Lamé coefficients λ and μ. These have an average and fluctuations. The simplest example of the theory is the two-dimensional case, consistent with the graph theory of fracture connectivity (Fig. 1).

During any increment in the overall strain, α, fluctuations in the distortion (β) are naturally position dependent and give rise to changes in stiffness according to a strain increment dependent law. Koenders (1997) discusses a number of cases for the evolution rules, but here the appropriate rule is the one in which the change in the Lamé coefficients is proportional to the isotropic strain so that the protolith becomes softer when expansion takes place and stiffer under compaction. This case has the pleasant property that it works on invariants. The description with Lamé coefficients implies isotropic material properties, and by letting the change be proportional to invariants, isotropy is preserved. The isotropic evolution rule takes the form:

$$\delta\lambda(\mathbf{x}) = c_1^{(\lambda)} \left(\alpha_{aa} + \beta_{aa}(\mathbf{x}) \right);$$
$$\delta\mu(\mathbf{x}) = c_1^{(\mu)} \left(\alpha_{aa} + \beta_{aa}(\mathbf{x}) \right). \tag{3}$$

What is required is an expression to link $\beta(\mathbf{x})$ to α via the fluctuations in the stiffness. Koenders (1997), modifying earlier work by Kröner (1967), found that such a link can be found in terms of Fourier transformed variables. In this way the evolution of the stiffness is readily expressed in harmonic intensity form. The phenomenon of structure formation for an isotropic evolution rule is manifest when a deviatoric global strain path is present. For the explicit case of a crustal protolith heated from below by intruding mafic magma, a local deviatoric strain path is highly likely.

Strain path during thermal expansion

We have applied the simple thermal conductivity calculation of Carslaw and Jaeger (1959) to a 500 m-thick vertical section of protolith, assuming an initial temperature difference ΔT between the intruding basaltic magma and country rock of 400°C (Fig. 4). It is apparent that after a time sufficiently long to support

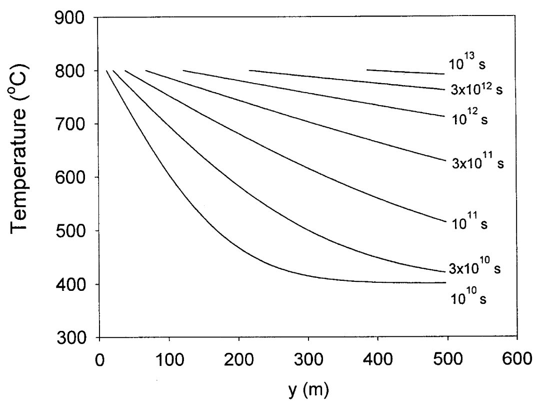

Fig. 4. Temperature profile as a function of position at different times in a one-dimensional calculation of melting and heat conduction in an infinite slab of rock (see Carslaw and Jaeger, 1959, 11.2). The following parameters have been used: temperature of intruding material 1000°C; melting point of rock 600°C; latent heat of the phase transition 1400 cal gm^{-1}; thermal conductivity of melt 0.001 cal/(cmsK); thermal conductivity rock 0.005 cal/(cmsK); thermal diffusivity of melt 0.01 cm^2 s^{-1}; thermal diffusivity rock 0.01 cm^2 s^{-1}; specific heat of melt 0.2 cal/(gmK). The profile of the temperature is shown here over a vertical protolith thickness of 500 m. The time taken to attain a more or less constant temperature is 3×10^{12}s (10^5 y). At this time about one third of the protolith is melted.

a no-resolidification state (ca. 3×10^{12} s), the protolith has undergone a more or less uniform increase in temperature. By this time, thermal expansion in the protolith is of the order of magnitude of several percent of volume strain. Estimating volume strain is complicated by the fact that dry rock and fluid (melt) filled fractures have different thermal expansion coefficients. However, there is no doubt that straining will take place on a scale which is slightly larger than the general tectonic background strain field of ca. 10^{-13} s^{-1}. A rough estimate made on the basis of the assumption that the melt phase will expand with an expansion coefficient of the order of 10^{-4} K^{-1} and the volumetric expansion coefficient of the solid rock is some 3×10^{-5} K^{-1}, the total expansion of the mixture under a temperature increase (ΔT) of 400 K is between 1% and 5% strain. Thus, in this case it is not the rate of strain but the *direction* of straining that dominates, giving rise to a deviatoric strain sufficient to cause alignment of fractures in the protolith.

Deviatoric strain and vertical fracture alignment

Fracture alignment is achieved in the following way. In the horizontal direction (y), the protolith is assumed to be clamped by the presence of cooler country rock that is not heated above its ambient temperature. It thus follows that all strain is approximately caused by vertical differential motion, denoted by the suffix x. The global strain path in the protolith is therefore deviatoric and has the form:

$$\begin{aligned} \alpha_{xx} &= \alpha_0 \\ \alpha_{12} &= \alpha_{21} = 0 \\ \alpha_{22} &\approx 0. \end{aligned} \tag{4}$$

The strain path so defined will be followed persistently in the protolith during thermal expansion.

An initial isotropic state is assumed which peaks around a certain wave number, corresponding to the heterogeneity scale. Typically this scale is the average distance between fractures in the protolith. This is not to imply that all fractures are equidistant, but that the process of fracture formation in the past history of the protolith has a certain, statistically defined, preponderance for the distance between fractures to be a particular size. Below this wave number there is no contribution as correlations die out for large distances. Concerted effects cannot take place on a scale smaller than a certain grain size (which is part of the basic properties of the rock) and therefore the harmonic intensity collapses for large $|k|$.

The distribution in shear modulus intensity represented by the harmonic intensity $S_\mu(\mathbf{k})$ for the protolith in the isotropic (unheated) state is shown in Fig. 5 as a function of wavenumber (Fig. 5a) and three dimensional form (Fig. 5b). The isotropic nature of the 'stiffness' is reflected in the fact that no directional features are present. The effect of deviatoric strain associated with heating and thermal expansion in the protolith after 10^5 y is shown in Fig. 6. As the overall strain increment [equation (4)] is repeatedly applied, the harmonic intensity becomes elongated (Fig. 6a). Changes in the harmonic intensity are more easily visualised in three-dimensional form (Fig. 6b) where directionality is manifest in the two distinct peaks in harmonic intensity, pointing to a vertically layered structure in the stiffness.

In summary, the direction of fractures in the protolith are normal to the direction in which the harmonic intensity is large, here the vertical (y) direction with structures in the spatial domain becoming elongated in the x-direction. This correlated behaviour resulting from evolution is equivalent to the coalescence of fractures in the vertical direction. In passing it is noted that the overall behaviour of the rock becomes anisotropic and also that the average moduli decrease under a persistent expansive strain path. These two effects together may give rise to bifurcative failure (e.g. Hill and Hutchinson, 1975).

DISCUSSION

While acknowledging that the discrete description of evolving fracture systems may at first sight appear artificial, this is not the case. To appreciate the importance of the emergence of order in fractured media for melt segregation it is necessary to consider the vast number of states in which the system is free to roam. Imagine a network made up of 100 randomly oriented fractures, and that each fracture exists in two possible states—melt filled (on) or melt-free (off). The number of possible states is then 2^{100}, a staggeringly vast number. Although it is true that this large number of states is theoretically available to the system, the choice of a suitable evolution process selects a state that enables melt segregation by creating directional structures. Some evidence for self-organised criticality in natural, high-level fracture systems is seen in the power law behaviour of acoustic emissions during volcanic activity (Diodati *et al.*, 1991). We argue that a similar process takes place in the lower crust during partial melting in the vicinity of a mafic heat source.

The continuum model provides a physical basis for the discrete model presented earlier. It shows that there exist necessary conditions for the evolution rule that lead to self-organisation. These conditions are: (1) a thermally-induced deviatoric persistent strain; and (2) a uniform temperature field during expansion. The latter condition is weaker than the strongly required former one. Thus, for fractures to align themselves vertically, the strain in this direction must be dominant. This happens because the country rock is effectively clamped in the horizontal direction due to lesser amounts of heating and thermal expansion. In this

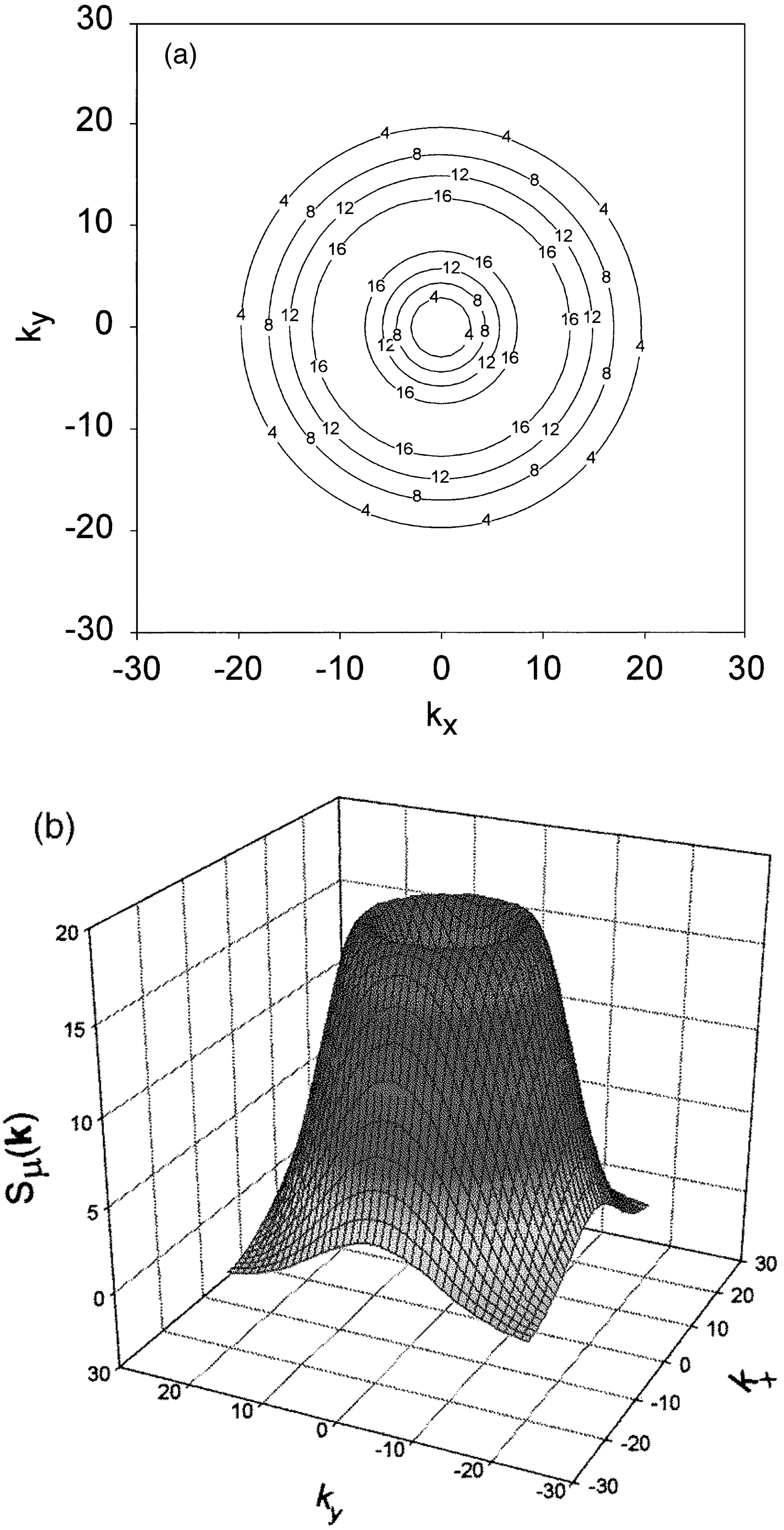

Fig. 5. (a) Contour plot showing the initial isotropic distribution of stiffness (shear intensity modulus) in the undeformed protolith. The 'stiffness' is shown as the Fourier transform of the correlation function as a function of the x and y components of the wavenumber vector **k**. (b) Three-dimensional form of Fig. 5(a) showing the undeformed harmonic intensity $S\mu(\mathbf{k})$.

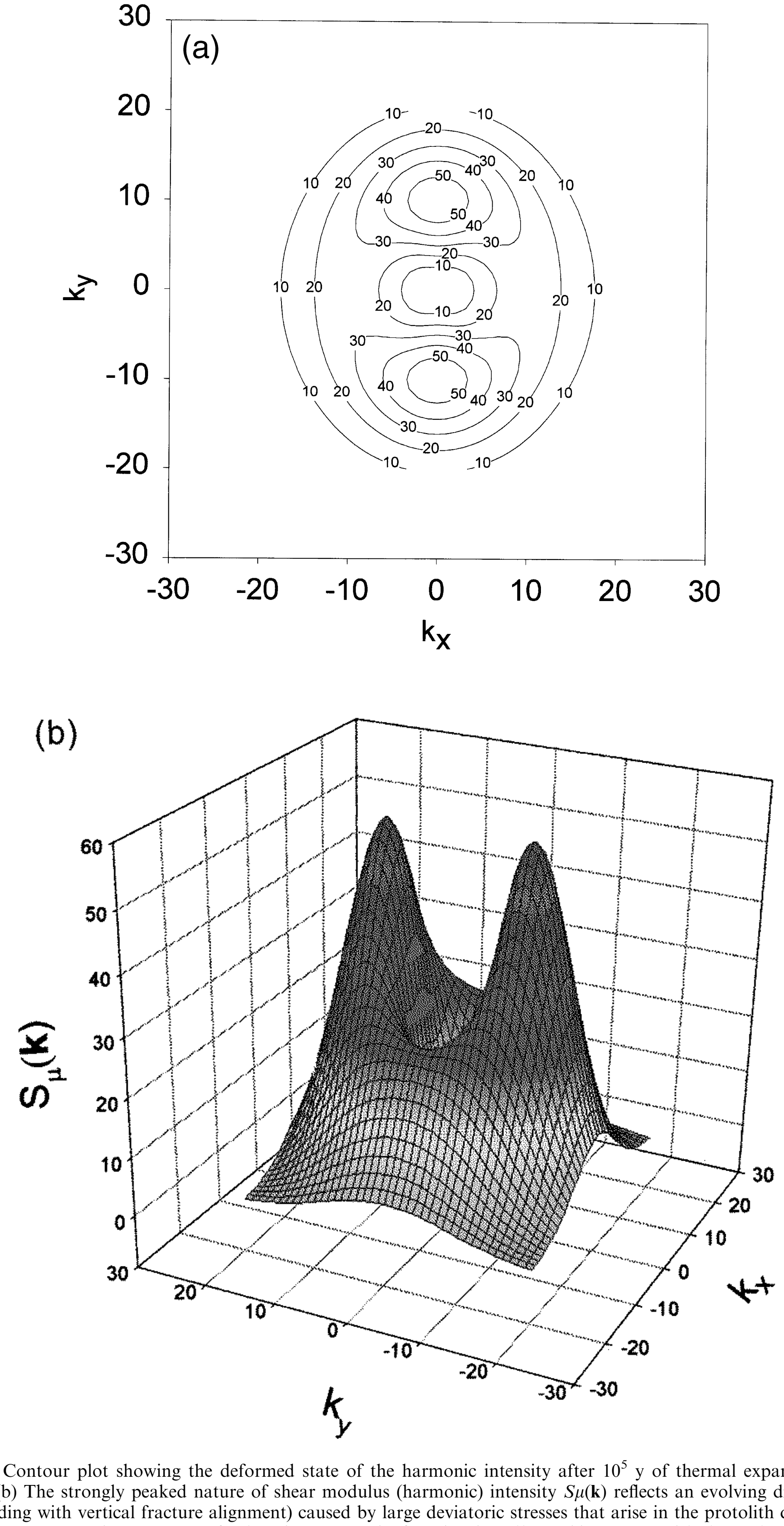

Fig. 6. (a) Contour plot showing the deformed state of the harmonic intensity after 10^5 y of thermal expansion in the protolith. (b) The strongly peaked nature of shear modulus (harmonic) intensity $S\mu(\mathbf{k})$ reflects an evolving directionality (corresponding with vertical fracture alignment) caused by large deviatoric stresses that arise in the protolith during heating. A more extensive sequence of colour images can be found at http://www.kingston.ac.uk/~gl_s041/npet.htm.

analysis, the time scale for heating, thermal expansion, self-organisation and vertical fracture alignment is of the order of 10^5 y. By this time ca. 30% of the protolith will be melted, consistent with chemical models of pluton formation. Our analysis suggests that partial melting during periods of crustal under/intraplating by mantle-derived magmas and subsequent fracture formation are strongly coupled 'pattern-forming' processes that act together to maximise the potential for melt to segregate effectively from its matrix.

CONCLUSIONS

Thermal forcing of crustal rocks during rapid heating (e.g. underplating) leads to the emergence of a self-organised fracture network. By analogy, fracture connectivity will develop locally from an initial uncorrelated state into a self-organised, communicating network (dissipative structure) that may focus and channel melt away from the source region. Predicted fracture connectivity increases rapidly across a relatively narrow transition region, effectively separating unconnected, isolated fractures from a highly connected fracture conformation. Provisional estimates of fracture permeability that take into account changes in connectivity range from ca. 10^{-10} m^2 to 10^{-5} m^2 at constant aspect ratio. Once fully connected, the fracture network forms a single, transient structure that will persist until either the heat flux into the protolith is exhausted, or the source region becomes thermally equilibrated. A physical continuum model is presented which predicts that changes in stress in a heterogeneous protolith characterised by stiffness domains will result in vertically oriented fractures in the source region over the timescale for thermal equilibration.

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